ME 105 – Mechanical Engineering Laboratory
Spring Quarter 2003

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5. VIBRATIONS

Introduction

It is not always possible to measure a quantity of interest directly. In this experiment, a spring-mass system, implemented as a cantilever beam with a tip mass, is allowed to oscillate freely. The position of the tip of the beam is determined with a sensor which measures strain and another which measures the acceleration. In each case a computation has to be carried out to indirectly determine the tip displacement; this is an indirect measurement. There is always some uncertainty in an experiment, and additional uncertainty is introduced by measuring quantities indirectly. A mechanical dial gage could be used to obtain a direct reading of this displacement. Such a gage, however, is useful only for static measurements and calibration.

Strain gages and accelerometers are sensors frequently used in mechanical measurements. This experiment provides an introduction to their use for dynamic measurements (i.e. measurements in which the observed quantities vary rapidly with time). The oscillation of the cantilever beam is monitored with an accelerometer mounted at the tip of the beam, and also with a pair of strain gages bonded to the beam at a point near its base. Issues related to the properties of the sensors, the electrical circuitry needed to condition their output, and the effects of various factors, such as temperature, on the measurements are examined.

Pre-Lab Reading

Thoroughly study this handout, and review relevant material from ME 163, ME 15, and your ME105 lecture notes.

Pre-Lab Work

Prepare a clear and organized outline of data acquisition for the session that includes tables with blank boxes to be filled with raw data and processed temperatures as well as brief reminders about what you will be doing and in what order. Make sure to look up and include at the appropriate place any property values, formulae, etc., that you might need during the session. It would be an excellent idea to include comments or (educated) guesses about what you think will happen or about trends you anticipate in you measurements, so that you can spot any problems or unusual results as they occur.

For the cantilever beam described in this handout, find the relationship between the tip displacement and the strain at the location of the gages. Prepare a sample calculation of the expected strain you would measure for 1 cm tip displacement.

Calculate the spring constant of the beam and find the undamped natural frequency of the beam assuming a 0.350 kg mass is attached at the tip. Calculate the maximum acceleration expected assuming the mass oscillates with constant 1 cm amplitude at the natural frequency. Make sketches of all five strain gage configurations you will use in the *bridge configurations* experiment describe later in this handout. You may neglect the balancing potentiometer. Write, for each case, the equation giving the change in output voltage in terms of the change in resistance of the strain gages. All resistances are assumed equal at the undeformed configuration.

All calculations and drawings should be available for inspection by the TAs at the beginning of the laboratory. The TAs will also pose questions on the handout and the above assignment to check preparedness.

Required Equipment

- flexible cantilever beam with tip mass attachments
- piezoelectric accelerometer with charge amplifier
- strain gages with Wheatstone bridge prototype board
- Digital Oscilloscope
- height gage

System Description

The system used in this experiment is shown in Figure 1. It consists of a cantilever beam with rectangular cross section attached to a heavy base, with a mass (a brass weight) attached to its tip. Strain gages are bonded to the upper and lower surface of the beam near the base. An accelerometer can be fastened to the tip mass. Some parameters of the system are:

E=197GPa	modulus of elasticity of beam
b=0.03m	beam width
h=0.0012m	beam depth
L=0.254m	beam length
L_s=0.0254m	distance of strain gage location from the root

The dimension given here should be verified and recorded with uncertainties in the laboratory. The added tip-mass and mass of the accelerometer are to be determined in lab.

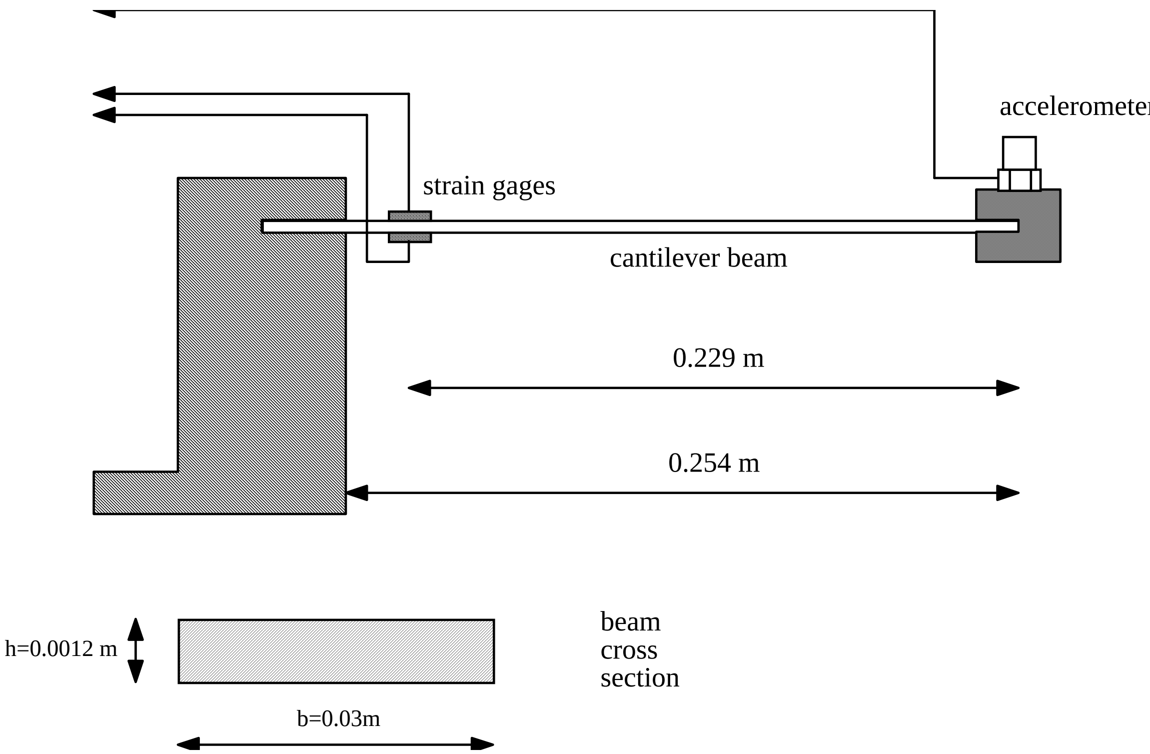

Figure 1: SDOF cantilever beam system

Oscillations of a SDOF System

The flexible beam with its tip mass constitutes a single-degree-of-freedom (SDOF) spring-mass oscillatory system, with damping due to dissipation of energy in the material of the beam. The equation of motion for free oscillation is:

$$m\ddot{y} + c\dot{y} + ky = 0 \qquad (1)$$

where m is the tip mass, c is the damping coefficient, and k is the cantilever beam spring constant. $y(t)$ is the displacement of the tip mass in the vertical direction from its equilibrium configuration. The equation can also be written as:

$$\ddot{y} + 2\xi\omega_n\dot{y} + \omega_n^2 y = 0 \qquad (1)$$

where:

$$\omega_n = \sqrt{k/m} \qquad (2a)$$

is the undamped natural frequency of the system, and:

$$\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{mk}} \qquad (2b)$$

is the (dimensionless) damping ratio, i.e. the ratio of the damping factor over the critical damping factor. Further the time constant of the system is defined as:

$$\tau = \frac{1}{\xi \omega_n} = \frac{2m}{c} \tag{3}$$

A convenient way to determine the amount of damping present in a system (expressed by the damping ratio or the time constant) is to measure the rate of decay of free oscillations of the system. Two methods are used: the "logarithmic decrement" method and the "half amplitude" method. In the first the damping coefficient is found from the logarithmic decrement, which is the natural logarithm of the ratio of the amplitudes of two successive peaks of the oscillation. In the second the damping coefficient is found from the number of cycles it takes for the amplitude of the oscillation to decrease by 50%. The damping ratio can also be determined from forced oscillations of the system, by quantifying the sharpness of resonance, using the so-called half-power points of the frequency response curve.

If the tip mass is given a displacement y_o at time t=0 (with zero initial velocity), the ensuing motion, found by solving (2), is:

$$y(t) = y_o e^{-t/\tau} \cos(\omega_d t) \tag{4}$$

where ω_d is the damped natural frequency:

$$\omega_d = \omega_n \sqrt{1 - \xi^2} \tag{4a}$$

In obtaining the solution in (4) we assumed that the system is underdamped ($\xi < 1$). If the system is very lightly damped ($\xi << 1$), the damped natural frequency is very close to the undamped:

$$\omega_d \cong \omega_n \tag{5}$$

The accelerometer directly measures the acceleration of the tip mass:

$$\ddot{y} = \frac{d^2 y}{dt^2} \tag{6}$$

Velocity and displacement are found by integration:

$$\dot{y}(t) = \dot{y}_o + \int_0^t \ddot{y}(t^*)dt^* \tag{7a}$$

$$y(t) = y_o + \int_0^t \dot{y}(t^*)dt^* \tag{7b}$$

The above material was covered in vibrations; review your textbook and class notes to resolve any questions prior to this experiment.

Mechanics of a Cantilever Beam

In order to determine the spring constant k and also to relate the strain at the base of the beam to the tip displacement we consider the mechanics of a cantilever beam subject to a transverse force applied at its tip, as shown in Figure 2. The bending moment at a cross section of the beam a distance x away from the root is found by constructing a free body diagram of the part of the beam to the right of the cross section at x:

$$M(x) = F(L - x) \tag{8}$$

The transverse deflection of the beam is calculated from the following differential equation:

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI} \tag{9}$$

E is the modulus of elasticity of the material and I is the cross section area moment of inertia:

$$I = \frac{bh^3}{12} \tag{10}$$

Using (8) in (9) and integrating twice we find:

$$y(x) = \frac{Fx^2}{EI}\left(\frac{L}{2} - \frac{x}{6}\right) + c_1 x + c_2 \tag{11}$$

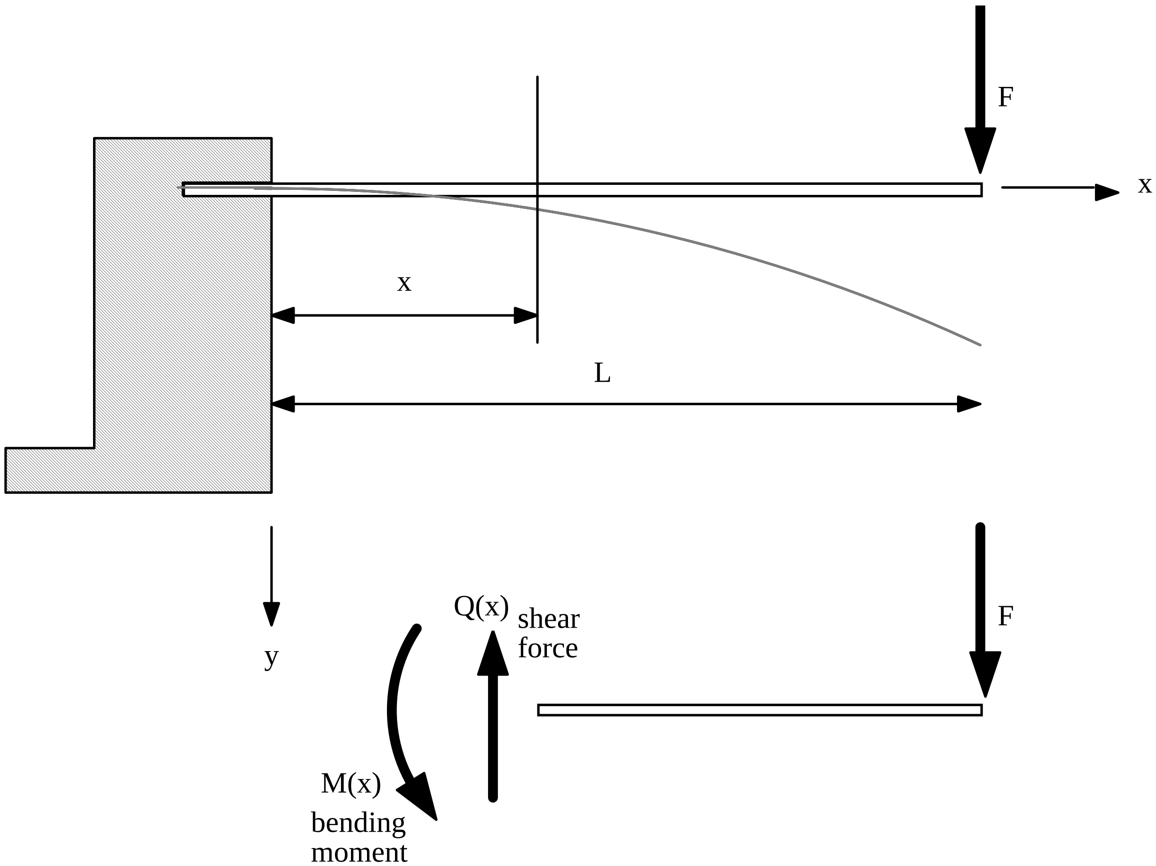

Figure 2: Bending of flexible beam

The constants of integration are determined using the boundary conditions at the root:

$$y(x=0)=0, \quad \left.\frac{dy}{dx}\right|_{x=0}=0 \quad \text{(clamped)} \tag{12}$$

The result is:

$$y(x)=\frac{Fx^2}{EI}\left(\frac{L}{2}-\frac{x}{6}\right) \tag{13}$$

The displacement at the tip is obtained from (13) by setting x=L:

$$y(x=L)=\frac{FL^3}{3EI} \tag{14}$$

Thus the spring constant (ratio of applied force over deflection) is:

$$k = \frac{F}{y(L)} = \frac{3EI}{L^3} \qquad (15)$$

The stress at the upper(lower) surface of the beam at a distance x from the root is:

$$\sigma(x) = \pm \frac{M(x)h}{2I} \qquad (16)$$

Tensile (resp. compressive) stress is considered positive (resp. negative). The strain is:

$$\varepsilon(x) = \frac{\sigma(x)}{E} \qquad (17)$$

The relationship between the tip displacement and the strain at the location of the strain gages (a distance L_s from the root) can be found from equations (8)-(17). This is left as an exercise. The above material was covered in ME 15; review your textbook and class notes to resolve any questions prior to this experiment.

Sensors and Other Hardware

Foil type strain gages are mounted on the upper and lower surface of the beam at a location 0.229 m from the tip (this should be verified). The (nominal) resistance of these gages, when undeformed, is 350 Ohm. The gage factor, F, relating strain to resistance is 2.07. The relationship between strain and relative change of resistance is:

$$\varepsilon = \frac{1}{F}\frac{\Delta R}{R} \qquad (18)$$

The strain, and thus the beam tip displacement, is determined from gage resistance changes using this equation. To determine the change in resistance we connect the strain gage(s) in a Wheatstone bridge and measure the output voltage. Two advantages are realized by using two gages connected in a bridge as shown in Figure 4. First, a two-gage bridge is relatively insensitive to temperature changes, and second, its sensitivity to strain changes is double that of a single-gage bridge. The Wheatstone Bridge Prototype Board to which the strain gages are connected allows simple reconfiguration and balancing. The output voltage from the Wheatstone bridge is established by the fixed resistor values (350 Ohm), the supply voltage (5 Volt), and the strain level being measured. The output of the bridge is connected to an oscilloscope.

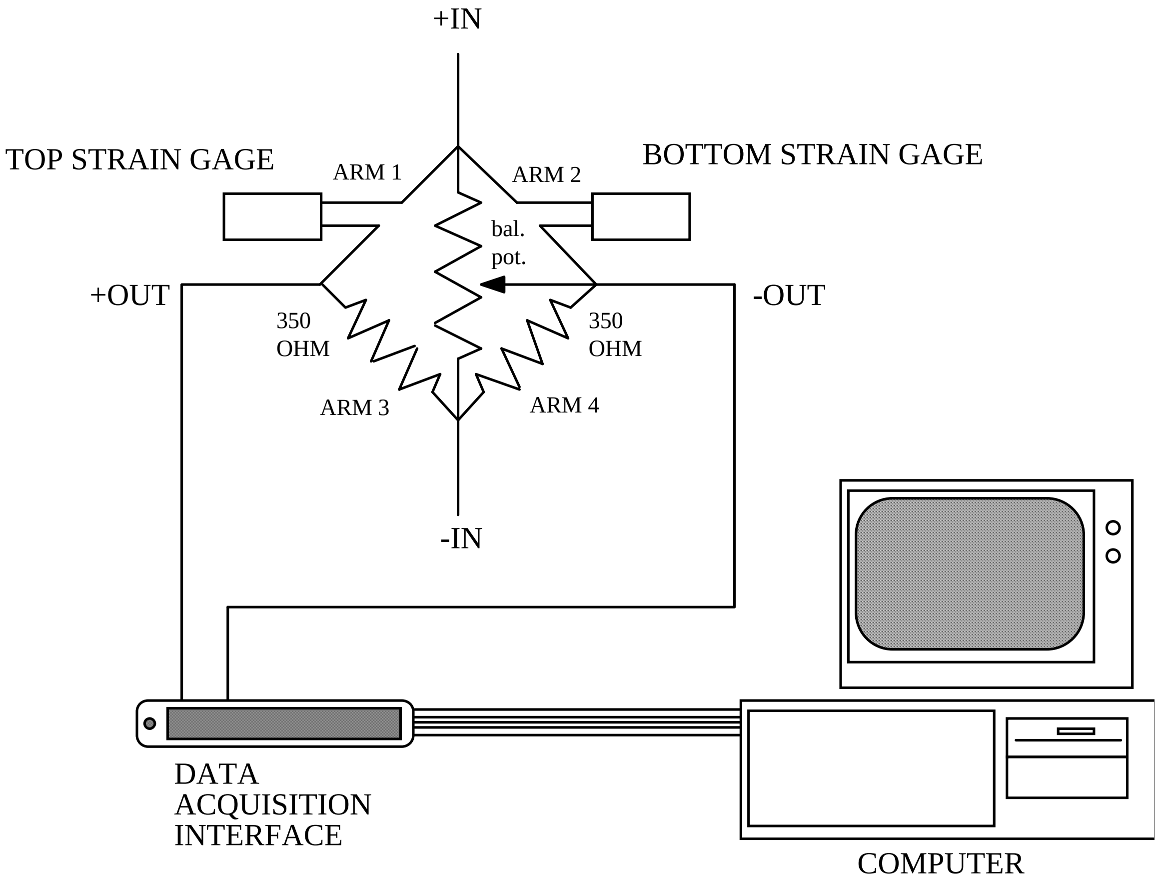

Figure 3: Wheatstone bridge

A piezoelectric accelerometer has as a sensing element a crystal mounted between the case and a proof mass allowed to move only in one direction. If the body to which the accelerometer is attached accelerates in the direction the mass is allowed to move, it causes the charge of the crystal to change by the piezoelectric effect. The applied force alters alignment of positive and negative ions in the crystal lattice structure, which results in an accumulation of charged on opposed surfaces. The total amount of accumulated charge is proportional to the applied force, and the applied force is proportional to acceleration. Electrodes collect and wires transmit the charge to a charge amplifier. The Endevco 7701-50 accelerometer used has a nominal sensitivity of 50 pCoulomb/g, g being the acceleration due to gravity. The Endevco 2721B, and similar charge amplifiers have a vernier potentiometer on the front panel, and the exact sensitivity of the particular accelerometer being used, as determined by calibration, is dialed in there. The gain of the amplifier is adjusted with the switch below the potentiometer and should be set to 300 mV/g. The output voltage of the amplifier is measured in mV/g and is connected to the data acquisition system.

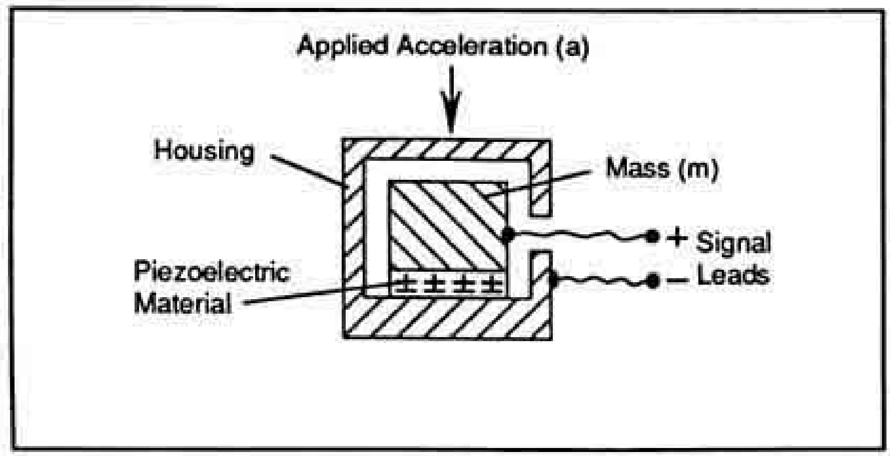

Figure 4: Piezoelectric Accelerometer

Experimental Procedure

Two channels of data will be displayed simultaneously on the screen of the oscilloscope. Instruction for saving data collected with the oscilloscope will be provided in the laboratory. Output from the charge amplifier used with the accelerometer should go to one channel; the output of the Wheatstone bridge should go to the other channel. For static measurements, you may want to measure the Wheatstone bridge output with a digital voltage meter.

Check the setup

Verify that the accelerometer is securely fastened to the large brass weight and that the charge amplifier is set correctly. Measure all dimension of the beam and the sensor locations. The gain of the charge amp should be set at 300 mV/g, and the dial should indicate the sensitivity found on the accelerometer calibration sheet attached to the workbench. Record this information.

Calibrate the strain gages

Three calibration weights are provided. Each has its mass (in gram) written on its side. Use each of the weights in turn, and calibrate the strain output versus known tip displacements. Use the height gage to find the displacements. The strain gages should be connected to arms 1 and 2 of the Bridge Prototype Board. Record all static strain-gage measurements and the corresponding weight values.

Experiment with the sensors

Place the large weight with the accelerometer attached to it on the beam. Set the indicated strain to zero using the bridge potentiometer. Make the tip mass oscillate by hand and observe the waveforms, paying attention to their relative phase. Twist the end of the beam and see if the strain gages are sensitive to torsional loads. Displace the beam and hold it in one position to observe the steady, or DC, response of both sensors. Holding a heat gun one foot from the sensors observe their response to temperature variations.

When heating the strain gages try to heat both the upper and lower one at the same time. Record your observations.

Observe the effect of different Wheatstone bridge configurations
Use the oscilloscope or the digital voltmeter to measure the voltage change which occurs in the strain output for the same displacement in the following configurations:

(**a**) Upper gage in arm 1, lower gage in arm 3
(**b**) Upper in 3, lower in 1
(**c**) Upper in 1, lower in 2
(**d**) Upper in 1, lower in 4
(**e**) Upper in 1, lower replaced with a fixed 350 Ohm resistor

Try to explain the results, using calculations you made in preparation for the experiment. It may be necessary to readjust the offset potentiometer in the center of the bridge to bring the display back within range as these changes are made. Remember that each arm of the bridge needs to have one jumper and one 350 Ohm load, either a fixed resistor or a strain gage. Leave the bridge in configuration (**e**) and again measure the temperature drift of a single strain gage. Record your observations.

Collect data from both sensors to compare estimates of tip displacement
Using the oscilloscope, record both the accelerometer and strain gauge response to free oscillations resulting from a moderately large initial tip displacement. Repeat this test until you have traces that clearly show the oscillation period and the decay of the oscillation. Be sure to save this data. You will use these results to estimate the system's damping ratio or its time constant.

Experiment Report

Give careful thought to a list of question that you feel are import to the results of this lab and that define a theme for your report. Annotated the report as you have been directed. In addition, your report needs to include a calculation of the tip displacement and the time constant using the data from both sensors taken when measuring the response to free oscillations. Compare these results and calculate of the uncertainty in your estimates of displacement.