TO: ME107A Consulting Group
FROM: George C. Johnson
RE: Vibrating beam analysis

Several years ago, one of my graduate students developed a set of resonant structures for the characterization and metrology of materials used in micro-electro-mechanical systems (MEMS). The basis for the structures is a relatively large end mass supported by a pair of parallel beams. A somewhat simplified version of this structure is shown in Figure 1. The beams, their support and the end mass are all made of polycrystalline silicon of thickness $t$. The beams themselves are of rectangular cross section, with width $w$, and the end mass has surface area $A$. This geometry provides flexibility in the direction normal to the beam axis, but is quite stiff in rotational motion of the mass. The beams are designed to move in the plane of the wafer.

We have derived a lumped parameter model for analyzing the natural frequency of the structures, but given the small size of these MEMS devices, testing is rather difficult. Therefore, we would like confirmation that the model is valid at the macro-scale.

Euler-Bernoulli beam theory suggests that the stiffness $k$ of each beam is

$$k = \frac{12EI}{L^3}, \quad I = \frac{tw^3}{12} \tag{1}$$

where $E$ is Young's modulus for the material and $I$ is the cross-sectional moment of inertia of the beam. The effective end mass $m_{eff}$ for the lumped parameter analysis involves not only the mass $m_{end}$ of the block at the end, but also some portion of the beam mass $m_{beam}$. An energy analysis provides

$$m_{eff} = m_{end} + 2\gamma m_{beam} \tag{2}$$

where, for this geometry, $\gamma = 13/35$ and the factor of 2 is present to account for the two beams. The undamped natural frequency of the system is then

$$\omega_n = \sqrt{\frac{2k}{m_{eff}}} \tag{3}$$

where, again, the factor of 2 is present to account for the two beams.

Our analysis does not allow us to predict in advance the damping that will be present in the actual system. However, we may want to use an electromagnetic damper that would allow us to increase damping. Further, we have not decided whether it will be more convenient to operate the system in free response or by providing harmonic excitation. Because of these uncertainties, you are asked to assess the damping and response characteristics of the beams in the lab.

The beams provided in the ME107A lab have the same basic geometry - a relatively large end mass supported by two parallel beams. A range of instruments have been provided on this test stand, including an accelerometer, a linear variable differential transformer (LVDT), a voice coil and fixed magnet for use as either a velocimeter or as a driver, and a pair of strain gages. Your assignment is to make measurements on the beam in the laboratory in order to assess the validity of the model presented above. In addition, you are to analyze the strength of the electromagnetic damping available with the laboratory system. The specific tasks to be performed are:

- Calibrate all of the relevant instruments: the LVDT, the accelerometer and the velocimeter. We will not use the strain gages this semester.
- Determine the dynamic stiffness and effective end mass of the system by adding masses to the end of the beam and analyzing it as a second order spring-mass-damper system. Are these experimentally determined quantities well predicted by Eqs. (1) and (2)?
- Provide a quantitative assessment of the amount of damping available from the voice coil and magnet when the coil is shorted. Is this amount of damping consistent with the calibration of the voice coil and magnet as a velocimeter?
- Compare the amplitude spectrum obtained by taking the fast Fourier transform of the free response of the system with that obtained by exciting the system at a range of frequencies. Your earlier work on creating a swept-frequency analyzer in HP-VEE might be useful in this regard.

Based on the above experimentation and analysis, please design a MEMS structure that has an undamped natural frequency of 40 kHz, assuming that the film thickness is $t$ = 2.0 μm, and that silicon has Young's modulus of $E$ = 170 GPa and mass density of $\rho$ = 2330 kg/m$^3$.

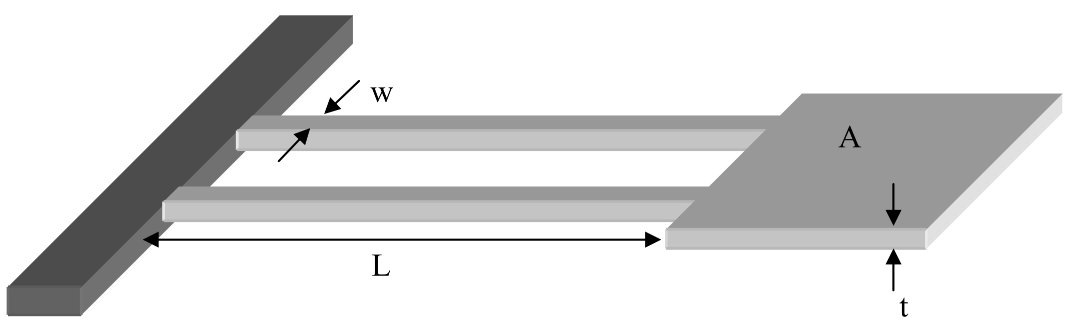


Figure 1. Schematic diagram of a resonant structure used for thin film material characterization. The structure is designed so that the mass on the right, of surface area $A$, moves in the plane of the material. The material on the left is rigidly attached to the substrate.

# Technical Data for Vibrating Leaf Springs

## SPRINGS

| Member | Material | Modulus (GPa) | Density ($kg/m^3$) | Thickness (mm) | Width (mm) | Length (mm) |
|---|---|---|---|---|---|---|
| Lower beam | 1060 steel | 205 ± 5 | 7780 ± 20 | 0.53 ± 0.02 | 25.4 ± 0.1 | 127 ± 1 |
| Upper beam | 1020 steel | 205 ± 5 | 7780 ± 20 | 0.59 ± 0.02 | 25.4 ± 0.1 | 127 ± 1 |

## STRAIN GAGES

| | |
|---|---|
| Resistance | 120Ω ± 0.2% |
| Gage Factor | 2.01 ± 0.5% |

## ACCELEROMETER

| | |
|---|---|
| Sensitivity (nominal) | 1 mV/($m/s^2$) |
| Resolution (nominal) | 0.2 $m/s^2$ |
| Range (±5%) | 1 – 10,000 Hz |

## LVDT

| | |
|---|---|
| Sensitivity (nominal) | 53 mV/mm/V at 3 kHz excitation |
| Linear Range | ±6.3 mm |

## ADDITIONAL MASSES

| | |
|---|---|
| Brass Washers | 13 ± 0.2 g |
| | 26 ± 0.2 g |
| | 52 ± 0.2 g |

Note: Web-based courseware is available for this lab at: http://bits.me.berkeley.edu/~beam/

It was written specifically for this apparatus, so it is likely to be VERY useful in understanding the set-up. It describes in some detail the theory and calibration of all of the instruments, and also covers free and forced response of the second-order system. The only problem might be playing some of the video clips and where links are provided to MathCad.